AQA Further Maths A-level

Mechanics

Formula Sheet

Provided in formula book	Not provided in formula book

This work by PMT Education is licensed under CC BY-NC-ND 4.0

Constant Acceleration

Motion in One Dimension

$s = ut + \frac{1}{2}at^2$
$s = vt - \frac{1}{2}at^2$
$v = u + at$
$s = \frac{1}{2}(u + v)t$
$v^2 = u^2 + 2as$

Motion in Multiple Dimensions

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$
$\boldsymbol{s} = \boldsymbol{v}t - \frac{1}{2}\boldsymbol{a}t^2$
$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$
$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$

PMT
resources • tuition • courses

Dimensional Analysis

Quantity	Dimension	SI Unit
Time	T	second (s)
Mass	M	kilogram (kg)
Length/Displacement	L	metre (m)
Area/Volume	L^2 / L^3	m^2/m^3
Velocity	LT^{-1}	ms^{-1}
Acceleration	LT^{-2}	ms^{-2}
Force	MLT^{-2}	newton (N)
Kinetic Energy	ML^2T^{-2}	joule (J)
Work Done	ML^2T^{-2}	joule (J)
Moment	ML^2T^{-2}	newton metres (Nm)
Power	ML^2T^{-3}	watt (W)
Momentum	MLT^{-1}	$kgms^{-1}$
Impulse	MLT^{-1}	newton seconds (Ns)
Moment of Inertia	ML^2	kgm^2
Angular Velocity	T^{-1}	rad s^{-1}
Frequency	T^{-1}	hertz (Hz)
Periodic Time	T	second (s)
Angle	1/Dimensionless	degree/radian
Density	ML^{-3}	kgm^{-3}
Pressure	$ML^{-1}T^{-2}$	pascal (Pa)

CC BY NC ND

Momentum and Collisions

Conservation of Linear Momentum

Momentum of an object of mass m moving at velocity v	$\text{momentum} = mv$
Momentum of an object of mass m moving with velocity vector $\begin{pmatrix} v_x \\ v_y \end{pmatrix}$	$\text{momentum} = m\begin{pmatrix} v_x \\ v_y \end{pmatrix}$
Conservation of momentum: Total momentum before collision = total momentum after collision	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Impulse, I, of a constant force F acting for time t	$I = Ft$

Restitution and Newton's Experimental Law

Coefficient of restitution, e	$e = \dfrac{v_2 - v_1}{u_1 - u_2}, 0 \leq e \leq 1$
Coefficient of restitution for a perfectly elastic collision	$e = 1$
Velocity, v, after a collision with a fixed object at initial velocity u	$v = -eu$

Defining Impulse as a Change in Momentum

Impulse needed to change the velocity of mass m from u to v (a change in momentum)	$I = mv - mu$

Impulse for Variable Forces

Impulse of a variable force $F(t)$ acting for a time t where $t_1 \leq t \leq t_2$	$I = \int_{t_1}^{t_2} F(t)dt$

CC BY NC ND

Work, Energy and Power

Definition of Work

Work done by a force acting in the direction of motion (unit: Joule, Newton Metre)	$\text{work done} = \text{force} \cdot \text{distance}$
Work done against/by gravity when raising/lowering a mass m through height h	$\text{work done} = mgh$

Gravitational Potential Energy

Gravitational potential energy (GPE) of an object of mass m at height h above ground level	$\text{GPE} = mgh$

Kinetic Energy

Kinetic energy of an object of mass m moving at a speed v	$\text{kinetic energy} = \frac{1}{2}mv^2$
Work done by a force on an object is equal to the change in its kinetic energy from moving with an initial velocity u to a final velocity v	$\text{work done} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
Conservation of mechanical energy	$\text{GPE} + \text{KE} = mgh + \frac{1}{2}mv^2 = \text{constant}$

Hooke's Law and the Modulus of Elasticity

Modulus of elasticity, λ, given by the ratio of the product of tension T and length l to the extension x	$\lambda = \frac{Tl}{x}$
Stiffness of an elastic string of length l and modulus of elasticity λ	$k = \frac{\lambda}{l}$
Hooke's law for an elastic spring or string of length l, modulus of elasticity λ, stiffness k, and extension x	$T = kx = \frac{\lambda x}{l}$
Work done extending an elastic spring or string from length x_1 to length x_2	$\frac{k}{2}(x_2^2 - x_1^2) = \frac{\lambda}{2l}(x_2^2 - x_1^2)$

CC BY NC ND

Work Done by a Variable Force

Work done by a variable force $f(x)$ acting on an object, moving it from position x_1 to x_2	$\text{work done} = \int_{x_1}^{x_2} f(x)dx$

Elastic Potential Energy

Elastic potential energy (EPE) stored in a string extended, or compressed, by length x	$\frac{kx^2}{2} = \frac{\lambda x^2}{2l}$
Conservation of energy for an object acted on by only its own weight and the force in an elastic spring or string	$\text{GPE} + \text{EPE} + \text{KE} = \text{constant}$

Power

Average power of a constant force applied for a given time (Unit: Watts, W)	$\text{average power} = \frac{\text{work done}}{\text{time taken}}$
Power in terms of tractive force	$\text{Power} = \text{tractive force} \cdot \text{speed}$

CC BY NC ND

Circular Motion

Angular Speed

Angular speed (rad s^{-1})	$\omega = \frac{d\theta}{dt}$

Kinematic Quantities in Circular Motion

Linear speed, v, of a particle moving in a circular path of radius r and with constant angular speed ω	$v = r\omega, \qquad \omega = \frac{v}{r}$
Centripetal acceleration, a	$a = v\omega = r\omega^2 = \frac{v^2}{r}$

Kinematic Quantities as Vectors in Circular Motion

For a particle with coordinates $\boldsymbol{r} = (r\cos\theta\,, r\sin\theta)$ where $\theta = 0$ when $t = 0$, with acceleration a and angular frequency ω:
$r = \begin{pmatrix} \mathrm{r}\cos\theta \\ r\ \sin\theta \end{pmatrix} = \begin{pmatrix} r\cos\omega t \\ r\sin\omega t \end{pmatrix}$
$\lvert a \rvert = \frac{v^2}{r}$

Conical Pendulum

For a particle to move in a horizontal motion (both for one, and two stringed pendulum):
$\text{Sum of perpendicular forces} = 0$
$\text{Resultant force towards centre of circle} = mr\omega^2$

Circular Motion in the Vertical Plane

Mechanical energy, the sum of kinetic and gravitational potential energy, is constant	$\frac{1}{2}mv^2 + mgh = const$
Condition for a particle connected to a string or rod to perform a full vertical rotation	Tension, $T > 0$ at the highest point of the vertical circle

CC BY NC ND

Centres of Mass and Moments

Centre of Mass of a System of Particles

Particles arranged in a straight line	$M\bar{x} = m_1x_1 + m_2x_2 + \cdots + m_nx_n$
Particles arranged in a plane	$M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m_1\begin{pmatrix}x_1\\ y_1\end{pmatrix} + m_2\begin{pmatrix}x_2\\ y_2\end{pmatrix} + \cdots + m_n\begin{pmatrix}x_n\\ y_n\end{pmatrix}$ $M = m_1 + m_2 + \cdots + m_n$

Centre of Mass of Composite Bodies

Centre of mass of a composite body	$M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m_1\begin{pmatrix}x_1\\ y_1\end{pmatrix} + m_2\begin{pmatrix}x_2\\ y_2\end{pmatrix} + \cdots + m_n\begin{pmatrix}x_n\\ y_n\end{pmatrix}$ $M = m_1 + m_2 + \cdots + m_n$

Centre of Mass of Uniform Laminae

Type of Body	**Centre of Mass**
Triangular lamina	$\frac{2}{3}$ along the median from the vertex, or the point of intersection of the medians $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
Solid hemisphere with radius r	$\frac{3r}{8}$ from centre
Hemispherical shell with radius r	$\frac{r}{2}$ from centre
Circular arc with radius r and angle 2α at the centre	$\frac{r\sin\alpha}{\alpha}$ from centre

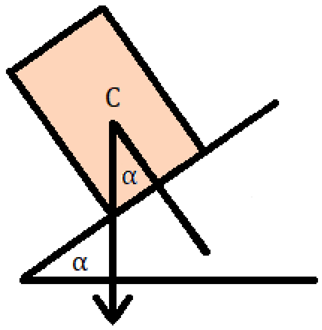
Sector of circle with radius r and angle 2α at the centre	$\frac{2r\sin\alpha}{3\alpha}$ from centre
Solid cone or pyramid with height h	$\frac{h}{4}$ from the base on the line between centre of base and vertex
Conical shell with height h	$\frac{h}{3}$ from the base on the line between centre of base and vertex
A lamina with a smaller piece removed	$M\binom{\bar{x}}{\bar{y}} = m_1\binom{x_1}{y_1} - m_2\binom{x_2}{y_2}$ $M = m_1 - m_2$
A uniform lamina defined by $f(x)$ and the lines $x = a, x = 0$ and $y = 0$:	$\text{Area} = \int_0^a f(x)\,dx$ $\bar{x} = \frac{\int_0^a x\,f(x)\,dx}{\int_0^a f(x)\,dx}, \bar{y} = \frac{\frac{1}{2}\int_0^a (f(x))^2\,dx}{\int_0^a f(x)\,dx}$

Centre of Mass Formed by Rotating a Region about the x-Axis

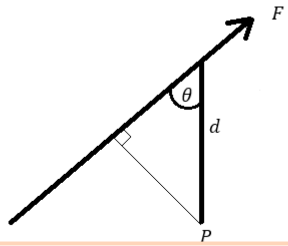
For a uniform solid of revolution with radius $f(x)$:
$\bar{x} = \frac{\int_0^a \pi x y^2\,dx}{\int_0^a \pi y^2\,dx}$
$\text{Volume} = \int_0^a \pi y^2\,dx$

Conditions for Sliding and Toppling on a Rough Inclined Plane

Suspension of a body from a point	A line drawn vertically down from the point of suspension will pass through the centre of mass
Condition for a body of mass m to slide down a rough plane with coefficient of friction μ inclined at angle θ to the horizontal	Component of the body's weight down the slope is greater than the frictional force: $mg\sin(\theta) > \mu mg\cos(\theta)$ $\Rightarrow \tan(\theta) > \mu$

Condition for a lamina to topple down a slope. The lamina will topple once the angle of inclination of the plane is larger than that between the line drawn from the edge of the line of contact to the centre of mass, with the vertical.	 Once the angle of the plane is larger than α, the lamina will topple. This lamina is on the point of toppling.

Forces Acting on a Rigid Body

Moment of force F about point P	$\text{Moment} = Fd\sin(\theta)$ 
Resultant moment from many individual moments	$M = \sum M_i = (r_i \times F_i)$
Resultant moment acting on a body in equilibrium	Zero resultant moment

CC BY NC ND